LITERATURE CITED

1. N. P. Tabunshchikov, Zh. Prikl. Khim., 29, No. 1, 32-35 (1956).
2. R. A. Kanova, Topical Sectional Correction from the Ministry of Ferrous Metallurgy of the USSR [in Russian], No. 3, 160-172 (1974).
3. R. A. Kanova, Topical Sectional Correction from the Ministry of Ferrous Metallurgy of the USSR [in Russian], 163-168 (1975).
4. K. M. Koval', All-Union Intercollegiate Collection on Hydrodynamics and Elasticity Theory [in Russian], Dnepropetrovsk (1981), pp. 34-40.
5. B. I. Kitaev, Yu. G. Yaroshenko, E. L. Sukhanov, et al., Blast-Furnace Heat Engineering [in Russian], Moscow (1978).
6. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Moscow (1970).
7. M. É. Aerov and O. M. Todes, Hydraulic and Thermal Principles in Equipment with Stationary and Fluidized Beds [in Russian], Moscow (1968).

A NONEQUILIBRIUM POLYDISPERSE FLOW IN AN AXISYMMETRIC NOZZLE WITH CONDENSATE PARTICLE COAGULATION AND BREAKUP

P. M. Kolesnikov and V. V. Leskovets

UDC 532.529

A kinetic description is given for the flow of a polydisperse medium, which is based on the general theory for nonequilibrium processes; the conversion to generalized transport equations is discussed and the closure of the infinite equation chain. An example is given of numerical calculations on such a flow in a nozzle on the basis of particle coalescence and breakup.

1. Theoretical studies may be made on polydisperse media by means of kinetic, statistical, or phenomenological methods. In the kinetic description of a system having variable mass m, speed **U**, temperature T, angular coordinates $\boldsymbol{\Theta}$, angular velocity, ω, angular momentum M, interaction force F, and other such parameters, one introduces the phase volume dΓ and a generalized 14-dimensional phase space.

$$d\Gamma = d\mathbf{r} d\mathbf{U} d\boldsymbol{\Theta} d\boldsymbol{\omega} dT dm$$

with the generalized distribution

$$f_i(t,\ \mathbf{r},\ \mathbf{U},\ \boldsymbol{\Theta},\ \boldsymbol{\omega},\ m,\ T).$$

We consider the motion and particle interaction in phase space and get the general kinetic equation there for the distribution:

$$\frac{\partial f_i}{\partial t} + \mathbf{U}\nabla_{\mathbf{r}} f_i + \nabla_{\mathbf{U}}\left(\frac{\mathbf{F}_i}{m_i} f_i\right) + \boldsymbol{\omega}\nabla_{\boldsymbol{\Theta}} f_i + \nabla_{\boldsymbol{\omega}}\left(\frac{\mathbf{M}_i}{I_i} f_i\right) + \nabla_T\left(\frac{\mathbf{q}}{m_i c_i} f_i\right) + \nabla_m\left(\frac{dm_i}{dt} f_i\right) = J_{ij}. \quad (1)$$

One of the most difficult aspects of kinetic theory is to determine the collision integral J_{ij} [1-4] in (1); if the medium is of sufficiently low density, it can be neglected [5-7]:

$$J_{ij} = 0. \quad (2)$$

Near thermodynamic equilibrium, that integral can sometimes be represented approximately as a relaxation relation

Lykov Institute for Heat and Mass Transfer, Belorussian Academy of Sciences, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 6, pp. 953-962, June, 1990. Original article submitted March 13, 1989.

0022-0841/90/5806-0741$12.50

$$J_{ij} = -\frac{f_i - f_{0i}}{\tau_{ij}}. \tag{3}$$

The next assumption is the diffusion approximation, where instead of a relaxation term, one introduces the second derivatives in parameter space by analogy with the Fokker-Planck approximation in ordinary kinetic theory [8-10].

The collision integral can also be derived by means of a procedure for deriving a hierarchic system of kinetic equations [4, 6, 8, 11], but then there are difficulties enclosing the infinite chain, which have so far been overcome only for the simplest thermodynamic systems [4].

When the kinetic equations have been formulated, one can examine the primary processes at the microscopic level and obtain very detailed information. Kinetic-theory methods have been applied to complicated systems such as low-density gases and plasmas [3-8], liquids [12], dispersed media [9, 10], reacting systems [13, 14], and ones showing phase transformations [15-17].

Extending the theory to nonequilibrium processes with any values for the parameters is a major aspect of kinetic theory. Although considerable results have been obtained from applying the theory to ideal and equilibrium systems, developments are still very much in hand for nonideal and nonequilibrium systems such as fluidized, rheological, turbulent, or self-organizing ones.

2. If a detailed description is superfluous, which is often the case for real measurements for observations on macroscopic processes, one can give an average description, where various statistical methods can be used, one of which is to average the kinetic equations describing the local behavior, where one ues rules and various additional conditions. Averaging the equations for molecular features leads to an infinite coupled chain.

One multiplies (1) by a molecular feature Ψ_i and integrates in the corresponding phase space to get equations for the average values of the molecular feature, which in kinetic theory are called transport equations for the molecular feature Ψ_i or generalized Maxwell–Enskog transport equations, as derived in gas kinetic theory [5]. If one averages (1) overall $\mathbf{U}$, $\boldsymbol{\omega}$, T, and m, one gets the generalized transport equation

$$\int_{-\infty}^{+\infty} d\mathbf{U} \int d\boldsymbol{\omega} \int dT \int dm \left[\frac{\partial f_i}{\partial t} + \mathbf{U}\nabla_{\mathbf{r}} f_i + \nabla_{\mathbf{U}}\left(\frac{\mathbf{F}_i}{m_i} f_i\right) + \boldsymbol{\omega}\nabla_{\boldsymbol{\Theta}} f_i + \nabla_{\omega}\left(\frac{\mathbf{M}_i}{I_i} f_i\right) + \nabla_T\left(\frac{\mathbf{q}}{m_i c_i} f_i\right) + \right.$$
$$\left. + \nabla_m\left(\frac{dm_i}{dt} f_i\right)\right] \Psi_i = \int_{-\infty}^{+\infty} d\mathbf{U} \int d\boldsymbol{\omega} \int dT \int dm J_{ij} \Psi_i. \tag{4}$$

If one takes for example the moments of the velocity, $\mathbf{U}^n$, where n = 0, 1, 2, ..., (4) gives an infinite system of coupled equations, some of which have the meanings of the conservation laws for mass, momentum, and energy, while others generate the closure equations. With such averaging in kinetic theory, one has a problem in obtaining a closed system sufficient to describe the coupled transport. Closing the chain of averaged moment equations is an unsolved problem at present. In kinetic theory, there are various systems of closure equations, which may be constructed from seven, 13, 21, or even 52 moments. Progress can be made here by integrating the chain directly by numerical or asymptotic methods in particular cases.

Distributions have to be determined by solving the kinetic equations, which is complicated even in the simplest case such as a Boltzmann equation.

The averaged systems give the fluxes of the molecular features (mass, momentum, energy, and so on), from which one can determine thermophysical parameters for comparison with experiment to judge the accuracy and other aspects of the closure system. Transport theory thus gives not only the equations but also the parameters apperaing there in phenomenological transport theory.

3. An example is a polydsperse medium in a channel, such as an axisymmetric two-phase flow in a Laval nozzle, where condensate droplets may coalesce or break up. If the conditions mentioned for example in [18] are meant, the flow can be described on a multivelocity multitemperature model for a continuous medium [19-23], in which the actual flow is replaced by the flow of several interpenetrating and interacting continuous media.

In [20], a system has been given for the motion of such a medium as a multivelocity continuum with exchange between fractions described by balance relations; if own gives detailed terms for the rates of mass, momentum, and energy transfer, one has system for the particular case of an axisymmetric flow in the Lagrange representation for fraction interaction [22]:

$$\nabla y\rho \mathbf{U}=0,\quad \nabla y\rho u\mathbf{U}+(yp)_x=y\sum_{i=1}^{l}\rho_i C'_{Ri}(u_i-u),$$

$$\nabla y\rho v\mathbf{U}+(yp)_y-p=y\sum_{i=1}^{l}\rho_i C'_{Ri}(v_i-v),$$

$$\nabla y\rho H_0\mathbf{U}=y\sum_{i=1}^{l}\rho_i\{C_{\alpha i}c_p(T_i-T)+C'_{Ri}[u_i(u_i-u)+v_i(v_i-v)]\},$$

$$p=\rho\frac{(k-1)}{k}\left[H_0-\frac{u^2+v^2}{2}\right], \tag{5}$$

$$\nabla y\rho_i\mathbf{U}_i=y\left[n_i\sum_{j=1}^{i}K'_{ij}\Phi_{ij}\rho_j-\rho_i\sum_{j=1}^{l}K'_{ij}\Phi_{ij}n_j\right],\ \nabla yn_i\mathbf{U}_i=-yn_i\sum_{j=i}^{l}K_{ij}\Phi_{ij}n_j,$$

$$\nabla y\rho_i u_i\mathbf{U}_i=y\Big\{\rho_i C'_{Ri}(u-u_i)+n_i\sum_{j=1}^{i}K'_{ij}\rho_j[u_j-(1-\Phi_{ij})u_i]-$$

$$-\rho_i\sum_{j=i}^{l}K'_{ij}n_j[u_i-(1-\Phi_{ij})u_{ij}]\Big\},$$

$$\nabla y\rho_i v_i\mathbf{U}_i=y\Big\{\rho_i C'_{Ri}(v-v_i)+n_i\sum_{j=1}^{i}K'_{ij}\rho_j[v_j-(1-\Phi_{ij})v_i]-\rho_i\sum_{j=i}^{l}K'_{ij}n_j[v_i-(1-\Phi_{ij})v_j]\Big\},$$

$$\nabla y\rho_i T_i\mathbf{U}_i=y\Big\{\rho_i C_{\alpha i}\frac{c_p}{c_{\text{d}}}(T-T_i)+\frac{1}{c_{\text{d}}}\Big[n_i\sum_{j=1}^{i}K'_{ij}\rho_j E_{ji}+ \tag{6}$$

$$+\rho_i\sum_{j=i}^{l}K'_{ij}(1-\Phi_{ij})n_iE_{ij}\Big]-T_i\Big[\rho_i\sum_{j=i}^{l}K'_{ij}\Phi_{ij}n_j-n_i\sum_{j=1}^{i}K'_{ij}\Phi_{ij}\rho_j\Big]\Big\},$$

$$E_{ji}=c_{\text{d}}(T_j-T_i)+\frac{1}{2}[(u_j-u_i)^2+(v_j-v_i)^2],\quad K'_{ij}=K_{ij}\mathrm{E}_{ij}.$$

The collision-performance factor is provided by the [23] expression, which incorporates the effects from the gas flow on the collisions between particles in fractions i and j:

$$\Phi_{ij}=1-0.247\,\mathrm{Re}_{ij}^{0.434}\mathrm{Lp}_j^{-0.133}(r_i/r_j)^{0.273}-$$
$$-0.18\,\mathrm{We}_j^{0.67}\mathrm{Re}_{ij}^{0.395}\mathrm{Lp}_j^{0.117}(r_i/r_j)^{2.27}. \tag{7}$$

The particle trapping coefficient E_{ij} is defined from [18]:

$$\mathrm{E}_{ij}=(\mathrm{E}_{\text{d}ij}+\mathrm{E}_{\text{p}ij}\mathrm{Re}_{ij}/60)/(1+\mathrm{Re}_{ij}/60),$$
$$\mathrm{E}_{\text{d}ij}=[1+0.75\ln(4\,\mathrm{Stk}_{ij})/(2\,\mathrm{Stk}_{ij}-1{,}214)]^{-2}, \tag{8}$$
$$\mathrm{E}_{\text{p}ij}=[(\mathrm{Stk}_{ij})/(\mathrm{Stk}_{ij}+0{,}25)]^2.$$

The resistance coefficient C_{Ri} is described by a system of correlation equations [24]: for subsonic flow

$$C_{Ri}=C_{R0i}\left[\left\{1+\frac{\mathrm{M}}{\mathrm{Re}}\left\{\mathrm{Re}+\mathrm{M}\sqrt{\frac{k}{2}}\left[4.33+\frac{3.65-1.53\,(T_i/T)}{1+0.353\,(T_i/T)}\times\right.\right.\right.\right.$$

$$\left.\left.\times\exp\left(-0.247\sqrt{\frac{2}{k}\,\frac{\mathrm{Re}}{\mathrm{M}}}\right)\right]\right\}^{-1}+\frac{\mathrm{Re}}{24}\{[4.5+0.38\,(0.03\,\mathrm{Re}+$$

$$+0.48\sqrt{\mathrm{Re}})]/(1+0.03\,\mathrm{Re}+0.48\sqrt{\mathrm{Re}})+0.1\mathrm{M}^2+0.2\mathrm{M}^3\}\times$$

$$\left.\times\exp(-0.5\mathrm{M}/\sqrt{\mathrm{Re}})+0.6\,\mathrm{M}\sqrt{\frac{k}{2}}\,[1-\exp(-\mathrm{M}/\mathrm{Re})]\right]; \qquad (9)$$

and for supersonic flow with $M \geq 1.75$

$$C_{Ri}=\left\{0{,}9+\frac{0{,}34}{\mathrm{M}^2}+1{,}86\sqrt{\frac{\mathrm{M}}{\mathrm{Re}}}\left[2+\frac{4}{k\mathrm{M}^2}+\right.\right.$$

$$\left.\left.+\frac{1{,}058}{\sqrt{k}\,\mathrm{M}}\sqrt{\frac{2T_i}{T}-\frac{4}{k^2\mathrm{M}^4}}\right]\right\}\Big/\left(1+1{,}86\sqrt{\frac{\mathrm{M}}{\mathrm{Re}}}\right).$$

In the range $1.0 \leqslant \mathrm{M} \leqslant 1.75$ C_{Ri} is derived by linear interpolation [24].

The heat-transfer coefficient $C_{\alpha i}$ is defined [25] on the basis of the Cavano correction for the effects of inertia and gas dilution:

$$C_{\alpha i}=C_{\alpha 0i}\,\mathrm{Nu}_i^0/[1+3{,}42\mathrm{M}_i\,\mathrm{Nu}_i^0/(\mathrm{Re}_i\mathrm{Pr})],$$

$$\mathrm{Nu}_i^0=2+0{,}459\,\mathrm{Re}_i^{0,55}\mathrm{Pr}^{0,33}, \quad C_{\alpha 0i}=1{,}5\mu/(\rho_{\mathrm{B}i}r_i^2\mathrm{Pr}).$$

One also incorporates the aerodynamic deformation in the droplets and the scope for breakup at the critical Weber number We_*. The deformation is incorporated by making corrections to C_{Ri} and in the expression for K_{ij} [23]:

$$C'_{Ri}=C_{Ri}\begin{cases}\exp(0.03\,\mathrm{We}_i^{1,5})=\beta_1, & \mathrm{Re}_s\geqslant 700,\\ (1+0.03\,\mathrm{We}_i)^2=\beta_2, & \mathrm{Re}_s\leqslant 150,\\ \beta_2+(\beta_1-\beta_2)/550\,(\mathrm{Re}-150), & 150\leqslant \mathrm{Re}_s\leqslant 700,\end{cases}$$

$$K_{ij}=\pi\,[r_i\,(1+0.03\,\mathrm{We}_i)+r_j\,(1+0.03\,\mathrm{We}_j)]^2\,|\mathbf{U}_i-\mathbf{U}_j|.$$

We consider the flow in a nozzle, in which the longitutinal speeds of the gas and droplets in the subsonic or transonic range and in the supersonic range (I and II correspondingly) are alwyas positive. For I, the system (5) for the gas is replaced by a nonstationary one (apart from the energy equation, which is hyperbolic along a current line x), while (6) remains stationary, since it is x-hyperbolic for $u_i > 0$. An iterative method [26] is used, which with known patterns for the parameters is used to integrate (6) and to compute and store the right-hand sides in (5). Then (5) is solved and the process is repeated until it converges with the necessary accuracy.

To integrate (5) by the collocation method, one uses a conservative MacCormac difference scheme with a second-order accuray [27]; one integrates (6) by means of an explicit-inexplicit second-order scheme.

The supersonic range is handeled by integrating (5) and (6) together, which here are of x-hyperbolic type; the calculated parameters are used at the right-hand boundary $x = x_1$ to region I to derive the initial data for the supersonic flow at $x = x_2$, where the Mach number at the axis of the nozzle is $M = 1.01$. The parameters on the line $x = x_1$ are derived by linear extrapolation from the internal nodes subject to the condition $x_2 < x_1$ to avoid any effect on the calculation in region II. At he left-hand boundary to the subsonic and transonic range, which coincides with the inlet section to the nozzle ($x = x_0$), three boundary conditions are set for the nostationarys ystem (5):

1) constant total enthalpy

$$H_0 = \frac{k}{k-1}\frac{p}{\rho} + \frac{u^2+v^2}{2};$$

2) constant entropy

$$S = p/\rho^k;$$

3) velocity vector direction

$$v/u = \frac{y}{Y(x)}\frac{dY(x)}{dx}\bigg|_{x=x_0}.$$

The initial flow is assumed to be in equilibrium and the droplet parameters are taken as equal to these for the gas, and

$$\rho(x_0, y) = \rho(x_0, y)\frac{z}{1-z}\varepsilon_i.$$

It is further assume that the droplets have log-normal distribution at the inlet with standard deviation $\sigma = 1.5$ and mean geometrical diameter $d_S = 1.1$ μm.

Symmetry conditions are imposed for (5) and (6) at the axis, while the no-penetration condition applies for the gas at the wall:

$$v(x, Y) = u(x, Y)\,dY(x)/dx,$$

while the attachment condition applies for the droplets.

The solution range for (5) is modified by introducing the new independent variables $\bar{x}=x$, $\xi=[y/Y(x)]^2$; this also causes the difference net to become more closely space towards the wall, which raises the accuracy in calculating the gas parameters in the narrow wall zone free from droplets. In those variables, (5) written as conservation laws becomes

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{G}}{\partial x} + 2\frac{\partial \mathbf{Q}}{\partial \xi} = \mathbf{M}, \tag{10}$$

in which

$$\mathbf{f} = Y^2(\rho,\ \rho u,\ \rho v)^{\mathrm{T}},\quad \mathbf{G} = (\rho u,\ \rho u^2 + p,\ \rho uv)^{\mathrm{T}} Y^2;$$

$$\mathbf{Q} = Y\sqrt{\xi}\,(v - \sqrt{\xi}\,Y_x u)(\rho,\ \rho u,\ \rho v)^{\mathrm{T}};$$

$$\mathbf{M} = Y^2\Big(0,\ \sum_{i=1}^{l}\rho_i C_{Ri}(u_i - u),\ \ \sum_{i=1}^{l}\rho_i C_{Ri}(v_i - v) + p/Y\sqrt{\xi}\Big)^{\mathrm{T}}.$$

We apply the MacCormac difference scheme to (10):

$$\hat{\mathbf{f}}_{ij}^{n+1} = \mathbf{f}_{ij}^{n+1} - \Delta t\left\{\frac{\mathbf{G}_{i+1,j}^n - \mathbf{G}_{ij}^n}{\Delta x} + 2\,\frac{\mathbf{Q}_{i,j+1}^n - \mathbf{Q}_{ij}}{\Delta \xi}\right\},$$

$$\mathbf{f}_{i,j}^{n+1} = \frac{1}{2}\left\{\mathbf{f}_{ij}^{n+1} + \mathbf{f}_{ij}^n - \Delta t\left[\frac{\hat{\mathbf{G}}_{ij}^{n+1} - \hat{\mathbf{G}}_{i-1,j}^{n+1}}{\Delta x} + 2\,\frac{\hat{\mathbf{Q}}_{ij}^{n+1} - \hat{\mathbf{Q}}_{i,j-1}^{n+1}}{\Delta \xi} - \hat{\mathbf{M}}_{ij}^{n+1}\right]\right\}.$$

The equations for the droplet motion are written along the droplet paths $y_{ij}(x)$ as conservation laws:

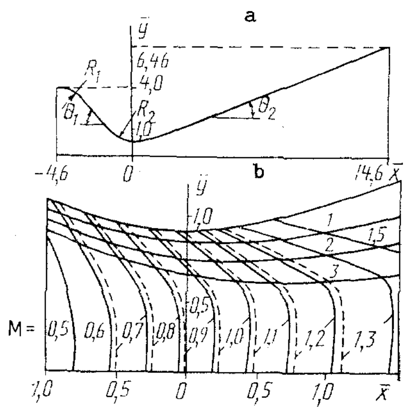

Fig. 1. Distribution of the Mach-number lines and positions of the limiting droplet lines for various diameters: 1) 1.07 μm; 2) 2.13; 3) 4.26 for z = 0.32 (b) in an axisymmetric nozzle (a).

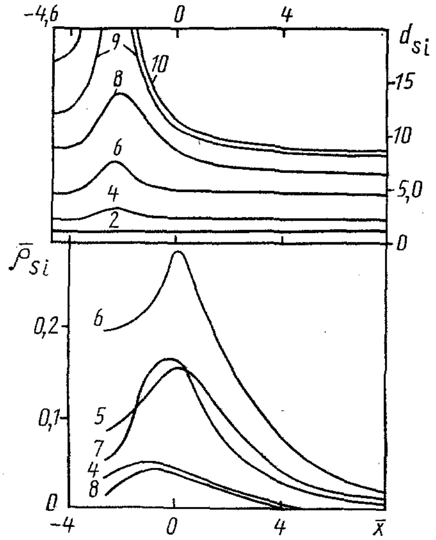

Fig. 2. Variation in condensate droplet diameter d_{Si} (μm) along a nozzle for fractions 2, 4, 6, 8, 9, and 10 and relative densities $\bar{\rho}_{Si}$ for fractions 4, 5, 6, 7, and 8 (the numbers are those of the fractions).

$$\frac{d\mathbf{A}_i}{dx}+E_i(\mathbf{A}_i-\mathbf{F}_i)+\mathbf{D}_i=0,\qquad \frac{dy_{ij}}{dx}=\frac{v_{ij}}{u_{ij}}, \tag{11}$$

in which

$$A_i=a\{\rho_i,\ u_i,\ n_iu_i,\ \rho_iu_i^2,\ \rho_iu_i,\ v_i,\ \rho_iu_iT_i\}^{\mathrm{T}};$$

E_i is a diagonal matrix having components

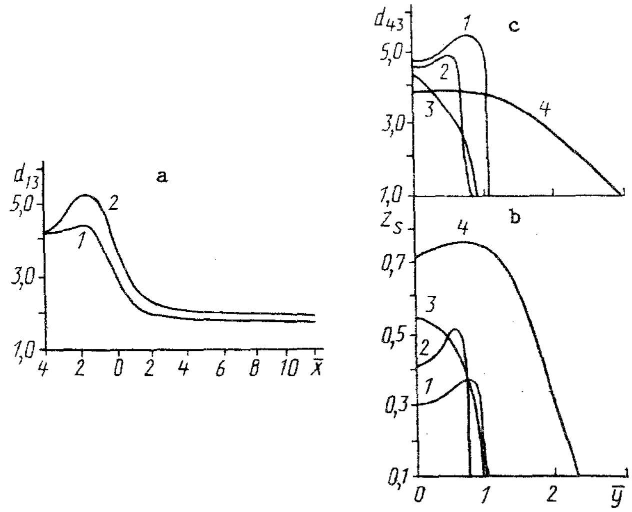

Fig. 3. Variations in mean mass size d_{43} in μm: a) along nozzle for z = 0.1 (curve 1) and z = 0.32 (curve 2); b and c) variations correspondingly in d_{43} and z_s across the nozzle in the sections x = −0.944 (curve 1); 0 (2); 1.34 (3); 8 (4).

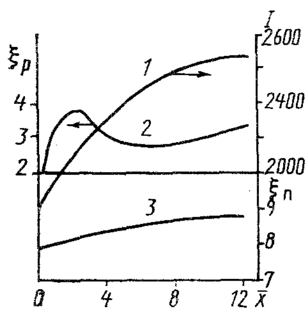

Fig. 4. Variation in specific momentum I in m/sec (curve 1), scattering loss ξ_s in % (curve 2), and two-phase factor ξ_n in % (curve 3) along nozzle.

$$\{0,\ 0,\ C'_{Ri}/u_i,\ C'_{Ri}/u_i,\ C_{\alpha i}/u_i\,(c_p/c_s)\};$$

$$\mathbf{F}_i = a\rho_i u_i\,\{0,\ 0,\ u,\ v,\ T\}^{\mathrm{T}},\quad a = y_{ij}\Delta_{ij},\ \Delta_{ij} = y_{ij} - y_{i,j-1};$$

and $\mathbf{D}_i$ are the coalescence terms from (6); j is the number of the droplet path. We write the explicit-inexplicit scheme [28] for (11) as

$$\hat{\mathbf{A}}_{ij}^{n+1} = [(\mathbf{A}_{ij}^n - \Delta x \mathbf{D}_{ij}^n)\,\Delta_{ij}^n + \Delta x E_{ij}^n\,\hat{\mathbf{F}}_{ij}^{n+1}]/[\Delta_{ij}^{n+1}\,(1 + \Delta x E_{ij}^n)],$$

$$\mathbf{A}_{ij}^{n+1} = \Big\{\mathbf{A}_{ij}^n \Delta_{ij}^n + \frac{\Delta x^2}{2} E_{ij}^n\,(\hat{E}_{ij}^{n+1}\,\mathbf{F}_{ij}^{n+1} - \hat{\mathbf{D}}_{ij}^{n+1})\,\Delta_{ij}^{n+1} +$$

$$+ \frac{\Delta x}{2}\,[\Delta_{ij}^n (E_{ij}^n \mathbf{F}_{ij}^n - \mathbf{D}_{ij}^n) + \Delta_{ij}^{n+1}\,(\hat{E}_{ij}^{n+1}\,\mathbf{F}_{ij}^{n+1} - \hat{\mathbf{D}}_{ij}^{n+1})]\Big\} \times$$

$$\times \left\{\Delta_{ij}^{n+1}\left[I + \Delta x E_{ij}^{n+1/2} + \frac{\Delta x^2}{2} E_{ij}^n \hat{E}_{ij}^{n+1}\right]\right\}^{-1},\quad E_{ij}^{n+1/2} = \frac{1}{2}\,(E_{ij}^n + \hat{E}_{ij}^{n+1}).$$

The calculations on two-phase flows were performed for radial-conical nozzles with parallel inlets (Fig. 1a); the nozzle contour in the minimal section is formed by an arc of a circle having radius R_2 = 2.01 (here and subsequently, all the dimensions are referred to the radius of the minimal nozzle section r_*), which has its center on the $\bar{y}$ axis and to which there are linked segments of straight lines whose inclinations to the axis are $\Theta_1=52°$ for the subsonic range or $\Theta_2=20°$ for the supersonic one. The radius of curvature is R_1 = 1.15 for the coordinates of the sections are shown in Fig. 1a. The nozzle contour is provided as a table, which is fitted to a bicubic spline with smoothing. The smoothing procedure has been described [29].

We used a set of 10 fractions, which in accordance with [30] was sufficient to provide the required accuracy.

The nozzle inlet data for model combustion products had [26]: T_0 = 3100 K, p_0 = 6 MP/m², ρ = 8.234 kg/m³, m = 26.2 kg/kmol, $\mu_0 = 8.937 \times 10^{-5}$ kg/m·sec, Pr = 0.69, c_d = 1876 J/kg·K, z = 0.32, 1 = 10, and σ_d = 0.4 N/m. The sizes of the droplets for fractions from one to 10 in the inlet section correspondingly were denoted by d_{si0} in μm of: 0.75; 1.07; 1.51; 2.13; 3.01; 4.26; 6.03; 8.53; 12.07; 17.07.

Figures 1-4 give some results for the basic form of nozzle with the above data unless otherwise mentioned; the general flow pattern in the neck region, where there are the highest parameter gradients, is shown in Fig. 1b. The solid lines are those of equal Mach number or the limiting lines for three fractions: curve 1 for d_{si} in μm of 1.07, 2 for 2.13, and 3 for 4.26, while the dashed lines are the similar ones for the gas without the second phase. As in [21-23], the sonic line in the expanding part of the nozzle is deflected as x increases, and one gets zones where there are small droplets at concentrations, whose width is dependent on the droplet growth rates in the subsonic part.

The droplet sizes are amongst the major parameters governing the integral characteristics; Fig. 2 shows how the diameters in the various fractions vary (the numbers are those of the fractions) along the nozzle. In the narrowing part, the droplets enlarge appreciably (the d_{si} in μm are given on the axis). The most pronounced changes occur for the large fractions, and in particular the diameters of fractions 8, 9, and 10 increase in the section x = 2 by factors of 1.66, 1.7, and 1.57 by comparison with the initial ones. The small and medium droplets do not change in size so substantially. The large fractions are much reduced in size in the transonic region because of rapid breakup. This occurs less rapidly in the exit section for the large fractions such as 8 and 10, where the sizes are reduced by factors of 1.4 and 2.07 by comparison with the initial ones. The sizes are relatively constant for x > 6, which is due to the reduced collisions because of the lower volume concentrations and lower density in the supersonic part. Figure 2 also shows the distributrion for the relative $\bar{\rho}_{si}=\sum_{i=1}^{l}\rho_{si}/\rho$ along the nozzle (the numbers are the fraction ones). The average over the cross section d_{43} (Fig. 3a) shows that there is a predominant tendency for the droplets to enlarge in the subsonic part and to become smaller in the transonic one and partially so in the supersonic one. As the mass proportion of the condensate falls (curve 2, z = 0.1), there is a less pronounced increase in the subsonic part, as the fractions interact less vigorously. The variations in $\bar{d}_{43}$ (in μm) in a series of sections may be seen from Fig. 3b (curve 1 for x = −0.94, 2 for 0, 3 for 1.34, and 4 for 8). Here the stepped d_{43} distribution corresponding to the 10 fractions has been smoothed, as is characteristic of an actual distribution. Figure 3c shows the same sections (curves 1-4) and gives the distributions for the local relative condensate flow rate

$$z_s=\sum_{i=1}^{l}\rho_{si}u_{si}\Big/\Big(\rho u+\sum_{i=1}^{l}\rho_{si}u_{si}\Big).$$

Figure 4 shows integral characteristics: specific momentum, loss due to two-phase structure ξ_n, and scattering ξ_s.

NOTATION

m_i, M, and c, mass, moment of inertia, and thermal capacity of droplet; J_{ij}, integral for the elastic and inelatic interactions between fraction i and fraction j; τ_{ij}, relaxation time; $\phi(\alpha)$, observed scattering indicatrix; f_{0i} equilibrium function [8]; u and v, components of velocity vector **U** along x and y axes; ρ, density; p, pressure; T, temperature; C_{Ri} and $C_{\alpha i}$, resistance and heat-transfer coefficients; H_0, stagnation enthalpy; k, adiabatic parameter; Y(x), nozzle contour; $Y_i(x)$, upper boundary to region occupied by fraction i; μ, dynamic viscosity; r, droplet radius; σ, surface tension; and speed of sound; K_{ij}, coalescence constant; Φ_{ij}, collision performance factor; E_{dij} and E_{pij}, particle trapping coefficients for viscous flow ($Re > 30$) and for potential flow in the region $Stk_{ij} > 0{,}1$; n_i, number of droplets in fraction i in unit volume (numerical concentration); ε_i, relative mass fraction for fraction i; z, mass fraction of condensate as a whole; M Mach number $M = |U_i - U|/a$; Re_{ij}, Reynolds number $Re_{ij} = 2r_i\rho_d|U_i - U_j|/\mu_{di}$; Nu_i, Nusselt number; Pr Prandtl number $Pr = c_p\mu/\lambda$; We Weber number $We = 2\rho_B U^2 r/\sigma_d$ Stk_{ij}; Stk_{ij}, Sotokes number $Stk_{ij} = |U_i - U_j|r_i^2\rho_j/9r_j\mu$; Lp_j, Laplace number $Lp_j = 2r_j\sigma_{dj}\rho_j/\mu_j^2$; subscripts s and d values relating to parameters and material of droplets correspondingly, subscript o parameter at stagnation point; n, i, and j inteters.

LITERATURE CITED

1. P. M. Kolesnikov and A. A. Karpov, Nonstationary Two-Phase Gas–Liquid Flows in Channels [in Russian], Minsk (1986).
2. P. M. Kolesnikov, Transport-Theory Methods for Nonlinear Media [in Russian], Minsk (1981).
3. L. Boltzmann, Lectures on Gas Theory [Russian translation], Moscow (1956).
4. N. N. Bogolyukov, Selected Works in Three Volumes [in Russian], Vol. 2, Kiev (1970).
5. S. Chapman and T. Cowling, The Mathematical Theory of Inhomogeneous Gases [Russian translation], Moscow (1960).
6. J. Ferziger and G. Kaper, The Mathematical Theory of Transport in Gases [Russian translation], Moscow (1976).
7. P. M. Kolesnikov, Electrodynamic Plasma Acceleration [in Russian], Moscow (1971).
8. Yu. Ya. Klimontovich, Statistical Physics [in Russian], Moscow (1982).
9. V. P. Myasnikov, Zh. Prikl. Mat. Tell. Fiz. , No. 2, 58-67 (1986).
10. Yu. A. Buevich, Prikl. Mat. Mekh., 35, No. 3, 464-481 (1971).
11. A. M. Gurov, The Principles of Kinetic Theory: Bogolyubov's Method [in Russian], Moscow (1966).
12. C. Crocksten, Liquid-State Physics: A Statistical Introduction [Russian translation], Moscow (1978).
13. B. V. Alekseev, The Mathematical Kinetics of Reacting Gases [in Russian], Moscow (1982).
14. S. V. Vallander, E. A. Nagnibeda, and M. A. Randalevskaya, Some Aspects of Kinetic Theory for Chemically Reacting Gas Mixtures [in Russian], Leningrad (1977).
15. Ya. I. Frenkel', The Kinetic Theory of Liquids [in Russian], Leningrad (1975).
16. V. P. Skripov, Metastable Liquids [in Russian], Moscow (1972).
17. V. A. Akulichev, Cavitation in Cryogenic and Boiling Liquid [in Russian], Moscow (1978).
18. L. E. Sternin, Principles of Gas Dynamics for Two-Phase Flows in Nozzles [in Russian], Moscow (1976).
19. R. I. Nigmatulin, Prikl. Mat. Mekh., 34, No. 6, 1097-1112 (1970).
20. A. N. Kraiko, R. I. Nigmatulin, V. K. Starkov, and L. E. Sternin, Surveys of Science and Engineering: Hydromechanics [in Russian], Vol. 6, Moscow (1972), pp. 127-153.
21. I. M. Vasenin, V. A. Arkhipov, V. G. Butov, et al., The Gas Dynamics of Two-Phase Flows in Nozzles [in Russian], Tomsk (1986).
22. A. D. Rychkov, Izv. Akad. Nauk SSSR, Mekh. Zhd. Graza, No. 1, 82-90 (1980).
23. L. E. Sternin, B. N. Maslov, A. A. Shraiber, and A. M. Podvysotskii, Two-Phase Monodisperse and Polydisperse Flows of Gas Containing Droplets [in Russian], Moscow (1980).
24. Henderson, Raket. Tekh. Komon., No. 6, 5-7 (1976).
25. Carlson and Hogland, Raket. Tekh., No. 11, 104-109 (1964).
26. Yu. F. Kisarov and A. M. Lipanov, Izv. Akad. Nauk SSSR, Mekh. Zhd. Graza No. 4, 161-165 (1975).
27. R. W. MacCormac, AIAA Paper, No. 354, 1-8 (1969).
28. A. D. Rychkov, Numerical Methods in Visocus-Liquid Dynamics [in Russian] (1983), pp. 86-92.
29. A. A. Blynskaya, Yu. B. Lifshits, V. D. Perminov, Uch. Zap. TsAGI, 5, No. 1, 128-131 (1974).
30. A. P. Tishin and R. I. Khairutdinov, Izv. Akad. Nauk SSR, Mekh. Zhd. Graza No. 5, 181-185 (1971).